Klee-Minty Polytope Shows Exponential Time Complexity of Simplex Method

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This is an example due to Klee and Minty [1] to show that the elementary simplex method does not have polynomial time complexity (worst case).

$$\begin{array}{rcccccccccl}
\max\ 2^{n-1}x_1 & + & 2^{n-2}x_2 & + & \ldots & + & 2x_{n-1} & + & x_n: & & \\
x_1 & & & & & & & & & \le & 5 \\
4x_1 & + & x_2 & & & & & & & \le & 25 \\
8x_1 & + & 4x_2 & + & x_3 & & & & & \le & 125 \\
\vdots & & & & & & & & & & \vdots \\
2^n x_1 & + & 2^{n-1}x_2 & + & \ldots & + & 4x_{n-1} & + & x_n & \le & 5^n \\
x \ge 0. & & & & & & & & & &
\end{array}$$

The LP has n variables, n constraints, and 2^n extreme points. The elementary simplex method, starting at $x = 0$, goes through each of the extreme points before reaching the optimum solution at $(0, 0, \ldots, 0, 5^n)$.

Pivot Sequence

Here is the pivot sequence for $n = 3$, which goes through all 8 extreme points, starting at the origin. Let s be the slack variables.

Initial Tableau

	Nonbasic			
Basic	x_1	x_2	x_3	RHS
s_1	1^*			5
s_2	4	1		25
s_3	8	4	1	125
$-z$	4	2	1	0
	↑			

Tableau 1

	Nonbasic			
Basic	s_1	x_2	x_3	RHS
x_1	1			5
s_2	-4	1^*		5
s_3	-8	4	1	85
$-z$	-4	2	1	-20
		↑		

Tableau 2

	Nonbasic			
Basic	s_1	s_2	x_3	RHS
x_1	1^*			5
x_2	-4	1		5
s_3	8	-4	1	65
$-z$	4	-2	1	-30
	↑			

Tableau 3

	Nonbasic			
Basic	x_1	s_2	x_3	RHS
s_1	1			5
x_2	4	1		25
s_3	-8	-4	1^*	25
$-z$	-4	-2	1	-50
			↑	

Tableau 4

	Nonbasic			
Basic	x_1	s_2	s_3	RHS
s_1	1^*			5
x_2	4	1		25
x_3	-8	-4	1	25
$-z$	4	2	-1	-75
	↑			

Tableau 5

	Nonbasic			
Basic	s_1	s_2	s_3	RHS
x_1	1			5
x_2	-4	1^*		5
x_3	8	-4	1	65
$-z$	-4	2	-1	-95
		↑		

Tableau 6

	Nonbasic			
Basic	s_1	x_2	s_3	RHS
x_1	1^*			5
s_2	-4	1		5
x_3	-8	4	1	85
$-z$	4	-2	-1	-105
	↑			

Tableau 7

	Nonbasic			
Basic	x_1	s_2	x_3	RHS
s_1	1^*			5
s_2	4	1		25
x_3	8	4	1	125
$-z$	-4	-2	-1	-125

References

[1] V. Klee and G.J. Minty. *How Good is the Simplex Algorithm?* In O. Shisha, editor, *Inequalities, III*, pages 159–175. Academic Press, New York, NY, 1972.